

### TRIUMPHS PSPs Available for Classroom Use (Fall 2022)

Descriptions of all PSPs available at: [https://nscoville.github.io/website/TRIUMPHS\\_project\\_descriptions.pdf](https://nscoville.github.io/website/TRIUMPHS_project_descriptions.pdf)

Complete versions of most of the PSPs listed below are available at: <https://digitalcommons.ursinus.edu/triumphs/>  
The *Notes to Instructors* section at the end of each PSP includes further information about its goals and design.

To obtain a preliminary copy of any PSP not yet posted on the TRIUMPHS website, contact: [janet.barnett@csupueblo.edu](mailto:janet.barnett@csupueblo.edu)

Full-PSPs (numbers correlate with posted PSP Descriptions)	Intended Course(s)	Author
F 01. A Genetic Context for Understanding the Trigonometric Functions <i>(Also available as six independent mini-versions; see M 41–M 46.)</i>	Pre-calculus /Trigonometry*	Danny Otero
F 02. Determining the Determinant	Linear Algebra	Danny Otero
F 03. Solving a System of Linear Equations Using the Ancient Chinese Methods	Linear Algebra *	Mary Flagg
F 04. Investigating Difference Equations	Discrete Mathematics	Dave Ruch
F 05. Quantifying Certainty: the p-value	Statistics	Dominic Klyve
F 06. Pythagorean Theorem and Exigency of Parallel Postulate	Geometry *	Jerry Lodder
F 07. Failure of the Parallel Postulate	Geometry	Jerry Lodder
F 08. Dedekind and the Creation of Ideals	Abstract Algebra	Janet Barnett
F 09. Primes, Divisibility & Factoring	Number Theory *	Dominic Klyve
F 10. The Pell Equation in Indian Mathematics	Number Theory *	Toke Knudsen & Keith Jones
F 11. Greatest Common Divisor: Algorithm and Proof	Introduction to Proof / Number Theory/ Discrete Mathematics /Abstract Algebra	Mary Flagg
F 12. The Möbius Function and Möbius Inversion	Number Theory	Carl Lienert
F 13. Bolzano on Continuity and the Intermediate Value Theorem	Introductory Analysis	Dave Ruch
F 14. Rigorous Debates over Debatable Rigor in Analysis: Monster Functions in Introductory Analysis	Introductory Analysis	Janet Barnett
F 15. An Introduction to Algebra and Geometry in the Complex Plane	Complex Variables	Diana White & Nick Scoville
F 16. Nearness without Distance	Topology	Nick Scoville
F 17. Connectedness - Its Evolution and Applications	Topology	Nick Scoville
F 18. Construction of Figurate Numbers	General Education *	Jerry Lodder
F 19. Pascal's Triangle and Mathematical Induction	General Education *	Jerry Lodder
F 20. The French Connection: Borda, Condorcet and the Mathematics of Voting Theory	Math for Liberal Arts / General Education *	Janet Barnett
F 21. An Introduction to a Rigorous Definition of Derivative	Introductory Analysis	Dave Ruch

\* indicates a PSP that is suitable for use in History of Mathematics Courses and/or Capstone Courses for Pre-service Secondary Teachers.

Full-PSPs (numbers correlate with posted PSP Descriptions)	Intended Course(s)	Author
F 22. Investigations into Bolzano's Bounded Set Theorem	Introductory Analysis	Dave Ruch
F 23. The Mean Value Theorem	Introductory Analysis	Dave Ruch
F 24. Abel and Cauchy on a Rigorous Approach to Infinite Series	Introductory Analysis	Dave Ruch
F 25. The Definite Integrals of Cauchy and Riemann	Introductory Analysis	Dave Ruch
F 26. Gaussian Integers and Dedekind Ideals: A Number Theory Project	Number Theory *	Janet Barnett
F 27. Otto Hölder's Formal Christening of the Quotient Group Concept	Abstract Algebra	Janet Barnett
F 28. Roots of Early Group Theory in the Works of Lagrange	Abstract Algebra	Janet Barnett
F 29. Radius of Curvature According to Christiaan Huygens	Vector Calculus	Jerry Lodder
F 31. Cross Cultural Comparisons: The Art of Computing the Greatest Common Divisor	Elementary Education Courses	Mary Flagg
F 32. A Look at Desargues' Theorem from Dual Perspectives	Geometry / Introduction to Proof	Carl Lienert
F 33. Solving Equations and Completing the Square: From the Roots of Algebra <i>(Also available in a mini-version; see M 28.)</i>	Pre-calculus *	Danny Otero
F 34. Argand's Development of the Complex Plane	Complex Variables *	Diana White & Nick Scoville
F 35. Riemann's Development of the Cauchy-Riemann Equations	Complex Variables	Dave Ruch
F 36. Gauss and Cauchy on Complex Integration	Complex Variables	Dave Ruch
F 37. Representing and Interpreting Data with Playfair <i>(Also available as three independent mini-versions; see M 31–M 33.)</i>	Statistics *	Diana White, River Bond, Joshua Eastes & Negar Janani
F 38. Runge-Kutta 4 (and Other Numerical Methods for ODE's)	Differential Equations	Adam Parker
F 39. Stitching Dedekind Cuts to Construct the Real Numbers	Introductory Analysis	Michael Saclolo
F 40. The Fermat-Torricelli Point and Cauchy's Method of Gradient Descent	Multivariable Calculus	Kenneth Monks
F 41. Stained Glass, Windmills and the Edge of the Universe: An Exploration of Green's Theorem	Multivariable Calculus	Abe Edwards
F 42. Finding Exact Sums of Infinite Series <i>(Available in two versions, a shorter version more suitable for Calculus 2 and a two-week version suitable for a capstone course.)</i>	Calculus 2; Capstone/enrichment courses *	Danny Otero & James Sellers
F 43. Deciphering the Calculations on Some Old Babylonian Tablets	Elementary Education *	Zoë Misiewicz
F 44: Fourier's Heat Equations and the Birth of Fourier Series <i>(Also available in a mini-version; see M 40.)</i>	Differential Equations / Multivariable Calculus	Kenneth Monks
F 45: Gauss and the First Rigorous Proof of the Fundamental Theorem of Algebra	Complex Variables / Introduction to Proof * (but could be used in any course above Calculus 2)	Sarah Hagen & Alan Kappler
F 46: Understanding Compactness Through Primary Sources: Early Work Uniform Continuity to the Heine-Borel Theorem	Introductory Analysis	Naveen Somasunderam

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Mini-PSPs (numbers correlate with posted PSP Descriptions)	Intended Course(s)	Author
M 01. Babylonian Numeration	General Education / Elementary Education Courses *	Dominic Klyve
M 02. L'Hôpital's Rule	Calculus *	Danny Otero
M 03. Derivatives of The Sine and Cosine Function	Calculus I *	Dominic Klyve
M 04. Beyond Riemann Sums	Calculus I *	Dominic Klyve
M 05. Fermat's Method for Finding Maxima and Minima	Calculus 1 *	Kenneth Monks
M 06. Euler's Calculation of the Sum of the Reciprocals of Squares	Calculus 2 *	Kenneth Monks
M 07. Braess' Paradox in City Planning: An Application of Multivariable Optimization	Multivariable Calculus	Kenneth Monks
M 08. The Origin of the Prime Number Theorem	Number Theory	Dominic Klyve
M 09. How to Calculate $\pi$ : Machin's Inverse Tangents	Calculus 2 *	Dominic Klyve
M 10. How to calculate $\pi$ : Buffon's Needle (Two versions available, one with no calculus pre-requisite.)	Calculus 2 ; Precalculus / Courses for Middle School Teachers *	Dominic Klyve
M 11. Bhāskara's Approximation and Mādhava's Infinite Series for Sine	Calculus 2 *	Kenneth Monks
M 12. Fourier's Proof of the Irrationality of $e$	Calculus 2 *	Kenneth Monks
M 13. Gaussian Guesswork: Elliptic Integrals and Integration by Substitution	Calculus 2	Janet Barnett
M 14. Gaussian Guesswork: Polar Coordinates, Arc Length and the Lemniscate Curve	Calculus 2	Janet Barnett
M 15. Gaussian Guesswork: Sequences & the Arithmetic-Geometric Mean	Calculus 2	Janet Barnett
M 16. The logarithm of $-1$	Complex Variables	Dominic Klyve
M 17. Why be so critical? Origins of Analysis in 19th Century Mathematics	Introductory Analysis *	Janet Barnett
M 18. Topology from Analysis: Making the Connection	Topology / Introductory Analysis	Nick Scoville
M 19. Connecting Connectedness	Topology	Nick Scoville
M 20. The Cantor Set before Cantor	Topology	Nick Scoville
M 21. A Compact Introduction to a Generalized Extreme Value Theorem	Topology	Nick Scoville
M 22. From Sets to Metric Spaces to Topological Spaces	Topology	Nick Scoville
M 23. The Closure Operation as the Foundation of Topology	Topology	Nick Scoville
M 24. Euler's Rediscovery of $e$	Introductory Analysis / Calculus 2	Dave Ruch
M 25. Henri Lebesgue and the Integral Concept	Introductory Analysis	Janet Barnett
M 26. Generating Pythagorean Triples via Gnomons (Two versions available, one more open-ended.)	Number Theory / Elementary Education Courses	Janet Barnett

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<b>Mini-PSPs (numbers correlate with posted PSP Descriptions)</b>	<b>Intended Course(s)</b>	<b>Author</b>
M 27. Seeing and Understanding Data	Statistics /General Education / Elementary Education Courses *	Beverly Wood & Charlotte Bolch
M 28. Completing the Square: From the Roots of Algebra	Pre-calculus *	Danny Otero
M 29. Euler's Square Root Laws for Negative Numbers	Complex Variables *	Dave Ruch
M 30. Investigations Into d'Alembert's Definition of Limit <i>(Two versions available, one lower-division, one upper-division.)</i>	Calculus 2 / Introductory Analysis	Dave Ruch
M 31. Playfair's Introduction of Bar and Pie Charts to Represent Data	Statistics *	Diana White, River Bond, Joshua Eastes & Negar Janani
M 32. Playfair's Introduction of Time Series to Represent Data	Statistics *	Diana White, River Bond, Joshua Eastes & Negar Janani
M 33. Playfair's Novel Visual Displays of Data	Statistics *	Diana White, River Bond, Joshua Eastes & Negar Janani
M 34. Regression to the Mean	Statistics *	Dominic Klyve
M 35. Solving Linear First Order Differential Equations: Gottfried Leibniz' Change of Variables	Differential Equations	Adam Parker
M 36. Solving Linear First Order Differential Equations: Johann Bernoulli's Variation of Parameters	Differential Equations	Adam Parker
M 37. Solving Linear First Order Differential Equations: Leonard Euler's Integrating Factor	Differential Equations	Adam Parker
M 38. Wronskians and Linear Independence: A Theorem Misunderstood by Many	Differential Equations	Adam Parker
M 39. Leonhard Euler and Johann Bernoulli on Solving Higher Order Differential Equations with Constant Coefficients	Differential Equations	Adam Parker
M 40. Fourier's Heat Equation and the Birth of Modern Climate Science <i>(Also available in a full-length version; see F 44.)</i>	Differential Equations / Multivariable Calculus	Kenneth Monks
M 41. The Trigonometric Functions Through Their Origins: Babylonian Astronomy and Sexagesimal Numeration <i>(See also F 01.)</i>	Pre-calculus /Trigonometry *	Danny Otero
M 42. The Trigonometric Functions Through Their Origins: Hipparchus' Table of Chords <i>(See also F 01.)</i>	Pre-calculus /Trigonometry *	Danny Otero
M 43. The Trigonometric Functions Through Their Origins: Ptolemy Finds High Noon in Chords of Circles <i>(See also F 01.)</i>	Pre-calculus /Trigonometry *	Danny Otero
M 44. The Trigonometric Functions Through Their Origins: Varāhamihira and the Poetry of Sines <i>(See also F 01.)</i>	Pre-calculus /Trigonometry *	Danny Otero
M 45. The Trigonometric Functions Through Their Origins: al-Bīrūnī Does Trigonometry in the Shadows <i>(See also F 01.)</i>	Pre-calculus /Trigonometry *	Danny Otero
M 46. The Trigonometric Functions Through Their Origins: Regiomontanus and the Beginnings of Modern Trigonometry <i>(See also F 01.)</i>	Pre-calculus /Trigonometry *	Danny Otero

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